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AN INVESTIGATION OF GROUPEMENT THEORY IN THE DOMAINS OF TIME AND SPEED

by

© NEIL WEINREB

A THESIS

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ABSTRACT

Certain predictions which are crucial tests of Piaget's groupement theory were examined in two concept areas (time and speed). In particular, those predictions are that all groupements are acquired in synchrony in a given concept area and that the constituent operations of a given groupement are acquired in synchrony. Piaget's theory of the development of time and speed concepts was also examined. Sixteen kindergarten children, 31 first graders, 31 second graders, and 31 third graders were tested on tasks which assessed the presence or absence of two groupements and their constituent operations in the time and speed concept areas, and which also assessed the presence or absence of the time and speed concepts themselves. There were four principal findings, the first two of which are inconsistent with groupement theory and the last two of which are inconsistent with Piaget's theory of time and speed: (*a*) The groupements do not emerge in concurrence in a given concept area; (*b*) the constituent operations of a given groupement do not emerge in concurrence; (*c*) the molar concepts of time and speed are grasped before all of their component operations are acquired; (*d*) the operations of speed are acquired before the operations of time. On the basis of these findings, groupement theory is challenged as a model for middle-childhood thought and Piaget's analysis

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of the development of speed and time concepts is challenged.

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INTRODUCTION

The purpose of the present study was to investigate certain aspects of Piaget's groupement theory (Brainerd, 1972; Flavell, 1963; Pinard and Laurendeau, 1969) and of his theory of the development of notions of time and speed[1] (Piaget, 1971a, 1971b). Basically, groupements are the cognitive structures which are models for the thought of the concrete-operational, or middle-childhood, period of development, and as such they are manifested in all concept areas. The two groupements which were studied are manifested in the time concept area as operations of temporal succession, or order, and operations of the colligation of durations, or temporal interval; in the speed concept area they are manifested as operations of spatial order and operations of the nesting of spatial intervals. For Piaget,

> "any representational act which is an integral part of an organized network of related acts is an operation (Flavell, 1963, p. 166)."

As is shown below, however, operations may be taken to refer to the set-theoretic operations of class union and intersection and the arithmetic operations of addition and multiplication.

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[1] Speed, rather than velocity, will be discussed, since the latter is composed of both speed and a vector component which is of no interest to the present study.

Piaget (1942, 1949) postulated that for any concept area all of the groupements and their constituent operations are acquired in synchrony. In the time concept area, then, it was postulated that the operations of temporal order and temporal interval are acquired in synchrony, and in the speed concept area, that the operations of spatial order and spatial interval are acquired in synchrony. Also according to Piaget (1971a, 1971b), operational, or molar, time and speed (*i.e.*, time and speed as they are understood by the child who is functioning fully at the concrete-operational level) are constituted through the coordination of all of the temporal and spatial operations.

Groupement Theory

A groupement is a synthesis of the group and lattice structures of abstract algebra (Brainerd, 1972; Flavell, 1963). Basically, a group is a set of elements combinable under a single operation and satisfying four specified postulates: Given a set *G* of elements, *a*, *b*, *c*, ... combinable under an undefined operation designated as "#." *G* is called a *group*, with respect to #, if the following postulates are satisfied:

1) Composition: For every *a*, *b* in *G*, *a*#*b* is also an element in *G*.
2) Associativity: For every *a*, *b*, *c* in *G*, (*a*#*b*)#*c*=*a*#(*b*#*c*).
3) Identity: There exists in *G* an element *I* (the identity element) such that for every *a* in *G*, *a*#*I*=*I*#*a*=*a*.
4) Reversibility: For every element *a* in *G*,

there exists an element a' in G such that $a\#a'=a'\#a=I$.

The lattice is a special case of a partially ordered system. Before this can be explicated, it must be made clear what a relation is: a *relation* between two elements can be said to exist whenever a specified propositional function R of two variables x, y is satisfied by these two elements. An example of a relation is "is greater than," and the expression xRy then means, "x is greater than y." Then, a set of elements S is said to be partially ordered with respect to a relation R if the following postulates hold on S:

1) R is reflexive: For every x in S, x has the relation R to itself, so that xRx is true.
2) R is antisymmetric: For every x, y in S, the conjunction (xRy and yRx) implies $x=y$.
3) R is transitive: For every x, y, z in S, the conjunction (xRy and yRz) implies xRz.

All that remains before "lattice" can be defined is a few more preliminary definitions. If x, y are elements (distinct or not) belonging to a partially ordered system S, and g is an element of S such that both gRx and gRy hold, then g is called a lower bound of x and y. If for any lower bound, g', of x and y, $g'Rg$ holds, then g is called a *meet* of x and y. If x, y are elements (distinct or not) belonging to a partially ordered system S, and h is an element of S such that both xRh and yRh, then h is called an

upper bound of x and y. If for any upper bound, h', of x and y, hRh' holds, then h is called a *join* of x and y. A *lattice* is a partially ordered system S in which every pair of elements x, y has a meet and a join. For example (Flavell, 1963, p. 172), if B is the class of mammals and it includes the class A of dogs, then A and B form a lattice in which R is the set-theoretic relation "subsumes (B subsumes A as a subclass)." The smallest class which includes both A and B is B, in this example the join of elements A and B. A, in this example, is the meet of the two elements, the largest class which is contained in both classes.

Piaget postulated four "class" groupements and four "relational" groupements (Brainerd, 1972; Flavell, 1963). The elements of the class groupements are classes and the composition operations are the set-theoretic operations of union and intersection, the reverse operations being the *inverses* of the composition operations (Brainerd, 1972). The effect of combining the lattice properties (the partial ordering postulates) with the group postulates and specifying the above operations is to produce structures which can be applied only to classes which are hierarchically ordered. For example, the class of dogs is nested in the class of mammals in the sense that there are no dogs which are not also mammals. The class of chihuahuas is similarly nested in the class of dogs, as is the class of

mammals nested in the class of vertebrates. If the elements of a class groupement are the class of dogs and the class of those mammals which are not dogs, then the union composition operation is the representational union of these two classes to yield the class of all mammals. The inverse operation is the representational removal of the class of dogs from the class of mammals to leave the class of mammals which are not dogs.

The elements of the relational groupements are relations and their composition operations are arithmetic addition and multiplication, the reverse operations being the reciprocals of the composition operations (Brainerd, 1972). The effect of combining the group postulates with the lattice and specifying the above operations is partially ordered systems which obey the group postulates as well as the lattice assumptions. For example, if an element of a relational groupement is the relation "<" and two operations performed on this element are ($\underline{A}<\underline{B}$) and ($\underline{B}<\underline{C}$), then the additive composition operation is the representational adding of ($\underline{A}<\underline{B}$) and ($\underline{B}<\underline{C}$) to yield ($\underline{A}<\underline{C}$). The result of the reciprocal operation is ($\underline{C}>\underline{A}$).

> "For purposes of conducting empirical research on the groupement structures, we need consider only the ... composition and reversibility properties ... (the) remaining properties are logical consequences of the first two. According to Piaget, the (other) properties also are developmental consequents of the first two properties (Brainerd, 1972, p. 6)."

Piaget postulated that within each groupement the composition and reverse operations emerge synchronously (Brainerd, 1972; Flavell, 1963). An operation is said to have emerged or been acquired as soon as its results are known by the cognizer without any actual physical manipulation of its contents. Piaget also postulated that all of the groupements emerge synchronously in any content domain (Brainerd, 1972; Flavell, 1963; Pinard and Laurendeau, 1969). A groupement is said to have emerged or been acquired as soon as both its composition and reversibility operations function simultaneously. Thus, the class groupement mentioned above can be said to have emerged as soon as the cognizer is able to unite simultaneously two subordinate classes (dogs and mammals which are not dogs) to form a superordinate class (mammals) and to subtract one subordinate class from the superordinate class to yield the remaining subordinate class. Also, the relational groupement mentioned above can be said to have emerged as soon as the knowledge that ($\underline{A}<\underline{B}$) and ($\underline{B}<\underline{C}$) simultaneously implies both ($\underline{A}<\underline{C}$) and ($\underline{C}>\underline{A}$).

Pinard and Laurendeau (1969), perhaps North America's most eloquent exponents of Piaget, have demonstrated the central role played by groupement theory by placing it in the perspective of five criteria they deem as essential to Piaget's stage theory in general. Hierarchization: a fixed order of succession exists among the various levels that

constitute a developmental sequence. *Integration*: "Asserts that the acquisitions of a given stage *S*2 should integrate those of the preceding stage *S*1, instead of simply substituting for them or juxtaposing with them (p. 127)." *Consolidation*: "A period must always involve at once an aspect of achievement of the recently acquired behavior and an aspect of preparation for the behavior of the following level (p. 129)." *Equilibration* is more difficult to characterize briefly but is, essentially, the mechanism of transition from one level of development to the next. The criterion of concern to the present study is that of *structuring*. According to this criterion, "The typical actions or operations of a given level are not simply juxtaposed one with another in an additive fashion, but are organically interconnected by ties of implication and reciprocal dependence that unite and group them into total structures--Piaget's *structures d'ensemble* (p. 136)." At the level of concrete operations the *structure d'ensemble* consists of the eight groupements interconnected, functionally interdependent, mutually implicative, etc. For Piaget, the very nature of the notion of *structure d'ensemble* implies the synchronous acquisition of the groupements, the gradual growth of interaction among them as they develop in concurrence. Also implied is the synchronous emergence of the composition and reversibility operations, not only within each groupement, but for all

groupements at once. This point is illustrated by the following:

> "To affirm the psychological existence of an authentic operational grouping bearing on a given content, this content must at the same time elicit not only the whole set of constituent operations of this grouping, but also the ensemble of parallel and connected groupings (p. 140)."

The present study was concerned with one class groupement and one relational groupement. Groupement I is that of the primary union of classes. The example above concerning classes of dogs and mammals and the operations performed on them is an instance of this groupement. The relational groupement is groupement V, that of the addition of asymmetric relations. The example above concerning the relation "<" and the operations performed on it is an instance of groupement V. (For an account of the other groupements, see Flavell, 1963.)

A distinction must be made between logical and infralogical operations. While they are analogous, there are some important differences. The former are applied to discrete, discontinuous objects which remain distinct and separate when combined into a whole; the latter are applied to wholes as single, continuous entities whose parts are not distinct when combined into wholes. The former are independent of the spatial and temporal proximity of the elements of their domain; the contents of the latter are

spatiotemporally continuous in character. Basically, infralogical operations are applied to the physical world of spatiotemporal wholes and parts, displacements of positions, etc., while the logical operations are applied to the realms of classes, number, etc. These differences, however, are not considered to alter the fact that infralogical groupements can be treated as being analogous to the logical groupements for most practical purposes (Flavell, 1963; Pinard and Laurendeau, 1969).

It will be shown that the operations of temporal succession and of spatial ordering are "grouped" to form structures which, in the infralogical world, correspond to groupement V. The operations of the colligation of durations and of the nesting of spatial intervals will likewise be shown to group into structures analogous to groupement I. A set of operations is said to have "grouped" when the composition and reversibility operations are simultaneously functioning.

Speed and Time

According to Piaget, all children conceive of speed at first as simply

> "... the intuition of overtaking: *A* travels faster than *B* (when both move in relation to *C* simultaneously and in the same direction) if *A*, which was at first placed to the rear of *B*, or at the same point, is finally placed ahead of it ... (thus) speed itself, as well as movement in general, is conceived at first in terms of relations of placing

(1971a, p. 313)."

Accordingly Piaget began his study of speed with a study of "placements," which are the positions of objects relative to each other and to the observer. An example of an operation of placement is the internalized arranging of a set of objects in some order, such as "*A* is to the left of *B*" and "*B* is to the left of *C*." The result of the additive composition of these two operations, itself an operation, is the knowledge that "*A* is to the left of *C*," and the result of the reciprocal operation is the knowledge that "*C* is to the right of *A*." These composition and reciprocal operations, which Piaget postulated to be acquired in synchrony (Piaget, 1971a), when functional simultaneously, are an instance of the grouping[2] of the addition of asymmetric relations, which is an embodiment of the properties of groupement V. Piaget stipulated another grouping of placements, that of the nesting of intervals: For example, the knowledge that the interval between *A* and *B* plus the interval between *B* and *C* together embody the interval between *A* and *C*, is the result of the composition operation; this operation, plus its inverse, the knowledge that the taking away of the interval between *A* and *B* leaves the interval between *B* and *C*, which Piaget also postulated to emerge in synchrony, is, functioning simultaneously, an

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[2] "Grouping" is the English translation of the French "groupement," and the meanings of the two are taken to be identical herein.

infralogical instance of the class union groupement, which is groupement I.

Operations of "displacement" are psychologically equivalent to operations of placement, according to Piaget, with the same groupings of asymmetric relations and nesting of intervals. The principal difference between the two is that placements are relative to the observer, while displacements are relative to a fixed coordinate system. Two functions of fixed coordinates are making the intervals between objects symmetrical by defining their end points and making the position of an object relative to its own initial position. When the relations of order among objects are considered along with the relations of each object to its initial position, and along with the notion of the symmetric interval, then the idea of interval is endowed with the meaning of distance, and interval can be regarded as distance travelled, or as path traversed. Thus, the concept of distance, in terms of which the molar forms of time and speed are defined, is, for Piaget, constructed when the grouping of the nesting of intervals is accomplished in correspondence with the grouping of the asymmetric relations of order in a fixed framework.

The operations of correlating two or more displacements in correspondence with one another are operations of "co-displacement." Concrete-operational speed is manifested at

the level of co-displacements:

> "A single displacement, and consequently a chain of displacements in turn, is a movement having no speed: whether *A* goes from *A* to *D* in one hour, one second, or at an unlimited speed, it is still the same displacement. There is thus no absolute speed in the sense of the speed of a movement in isolation ... on the other hand, if the successive positions of one moving object are ordered in relation to those of another object, the concept of speed necessarily intervenes, and this is in fact how it appears from the point of view of its psychological origins. For young children, speed is 'overtaking,' that is, the reversal of the order of the respective positions of two moving objects in the course of a displacement (Piaget, 1971a, p. 237)."

To illustrate, given a set of objects in a definite order *A*1*B*1*C*1*D*1 which are then displaced to *B*1*C*1*A*1*D*1, this is a displacement with no speed. Now, given objects in order *A*2*B*2*C*2*D*2 set in one-to-one correspondence with the original placements *A*1*B*1*C*1*D*1, this is a static set of placements which can be called a "state," state I. Then, given a state II with the placements *B*1*C*1*A*1*D*1 placed opposite *B*2*C*2*D*2*A*2, this is the same as saying that the displacement of *A*1 corresponds to a greater displacement of *A*2 relative to their respective initial positions. *A*1 and *A*2 were once "even," and now the position of *A*2 is ordered ahead of the position of *A*1 in the direction of movement. Thus, there has been an overtaking, a change in the relative positions of two objects with respect to some fixed coordinates (their respective initial positions), and, hence, the idea of speed

is introduced (A2 went faster than A1).

When the child is capable of performing operations of co-displacement, then, he must also, according to Piaget, have a grasp of the notion of speed. The speed of one object may be recognized as being greater or less than that of another according to the degree of overtaking. In addition, distances covered by moving objects between two states can be compared, and every overtaking will be accompanied by an inequality in paths traversed (providing, of course, that the moving objects started simultaneously and from corresponding points--the simplest case); the greatest speed can then be recognized by the fact that a longer distance has been travelled in a given time (between two states). Also, when operations of co-displacement are possible, a temporal order of succession is recognized in the sense that state II is after state I. This temporal order is seen to be distinct from spatial order, whereas the two kinds of order are confounded in the cases of placements and displacements: An object that is "ahead (behind)" of another in the spatial sense will also be "before (after)" it in the temporal sense in cases of placements and displacements, while in the case of co-displacements, with their distinct temporal states and several objects moving simultaneously, this confounding does not occur. For example:

> "... if A went further than B (A and B being objects travelling in the same direction along parallel paths, having started simultaneously from corresponding points), but B was still moving after A had stopped, (then) the temporal 'before' and 'after' are no longer identical with the spatial (Piaget, 1971a, p. 286)."

Finally, duration is recognized as the temporal interval between two states, between the limits of time elapsed, as opposed to being confounded with the spatial interval between initial and stopping positions of an object undergoing a displacement.

Thus it is seen that, according to Piaget, the notion of speed is derived from observations of overtaking, the latter being possible only during co-displacements. Co-displacements are coordinations of two or more displacements arranged simultaneously, and displacements are psychologically equivalent to placements. The two groupings of placements (those of spatial interval and spatial order), then, are the necessary basic components of Piaget's notion of speed at the concrete-operational level.

Since the notions of speed and time are so mutually bound up, it is not surprising that the two aspects of time which were investigated in the present study have already been introduced in the discussion of speed. The composition and reciprocal operations of temporal succession, which Piaget (1971b) postulated to emerge in synchrony, are, when functional simultaneously, an instance of the additive

grouping of asymmetric relations (groupement V). In its simplest form, this is transitive inference and its reciprocal performed representationally on a sequence of nonsimultaneous events. For example, knowing simultaneously that, if state _A_ occurs before state _B_ and if state _B_ occurs before state _C_, then state _A_ occurs before state _C_ and state _C_ occurs after state _A_, presupposes the acquisition of groupement V.

Duration, or temporal interval, is the "amount" of time elapsed between two nonsimultaneous events. The grouping of the colligation of durations is analogous to the grouping of the nesting of spatial intervals, an embodiment of the properties of groupement I. As temporal order becomes gradually dissociated from spatial order, so is duration gradually differentiated from the path traversed by a moving object.

Piaget believes, as mentioned above, that the operations of spatial order and spatial interval are grouped synchronously. The case is somewhat different for the temporal groupings: About half of the children tested by Piaget grouped the operations of duration first and derived the grouping of the operations of temporal order from them (instantaneously, so it seems), and _vice versa_ for the remainder of the children (Piaget, 1971b). Nevertheless, for the purposes of this study the two temporal groupings

can be regarded as emerging synchronously, that is, there is no general tendency for one grouping to be acquired before another. In addition, Piaget postulated that the two spatial groupings and the two temporal groupings are acquired in synchrony.

The "mature" concrete-operational, or molar, conception of speed is, for Piaget (1971a), qualitative: One moving object is known to have moved faster (or slower) than, or at the same speed as, another by the taking into account of the distances they travelled related to the time it took each object to do so, provided that the disparity in their distances or times or both is sufficiently detectable; but at the level of concrete operations it cannot be known _how much_ faster one object has travelled than another. If one object is travelling, or has stopped, ahead of the other, then operations of both spatial order and spatial interval are involved. The case of time is analogous (Piaget, 1971b): One moving object is known to have taken less (more) time than, or the same time as, another by the taking into account of the distances they travelled related to their speeds, again provided that the disparity in their distances or speeds or both is sufficiently detectable; it cannot be known _how much_ more or less time one object has taken. If one object has taken longer than another, then operations of both temporal order and duration are involved. It is not the case, then, that speed=distance/time and

time=distance/speed, for these formulas are quantitative; rather, speed=(distance)X(-time) and time=(distance)X(-speed) are the more correct expressions, where the minus signs are not to be taken as such, but rather as indications that time and speed, respectively, are inversely related to distance. These molar concepts of time and speed were postulated to emerge in concurrence as soon as operations of co-displacements are grouped (Piaget, 1971a, 1971b). It is not until the child is capable of formal-operational thought and the use of reiterable units for measurement that the concepts of time and speed can be said to take their Newtonian forms.

Previous Research

Theoretical. Pinard and Laurendeau (1969) have argued that the finding of asynchronous emergence of groupements in a given concept area would amount to a disconfirmation of the hypothesis of *structures d'ensemble*, the hypothesis that the operations of a given level of development are united into tightly interwoven total structures, and also

> "... would seriously jeopardize Piaget's conception of stage because it would deny one of its most essential characteristics, and because it would be difficult to reconcile with the very nature of groupings (p. 145)."

These authors further postulated that any observed asynchrony among groupements or operations applying to a single conceptual domain must result from nonconceptual

differences in the tasks used to study these structures, rather than from competence differences. For example, success in solving a transitive inference problem with a series of seven objects is sure to come later, due to information-processing, perceptual, etc., deficiencies, than success on the same problem with three objects, although the cognitive structure being assessed is the same in both instances.

Brainerd (1972) took a similar stand concerning the possible finding of asynchronous emergence of groupements and operations. According to him, synchronous emergence is such a crucial empirical prediction of groupement theory that the failure to find synchrony would seriously jeopardize its claim as a model for middle-childhood thought.

According to Flavell and Wohlwill (1969), on the other hand, there is no logical reason to consider the groupements as so interdependent as to preclude their being treated as separate entities, each following its own developmental timetable. They do not believe that such asynchrony would be seriously vitiating for Piaget's groupement theory, but that it might rather be instrumental in requiring a more precise specification of the entire concrete-operational period.

Flavell (1970, 1971) further suggested that the

interdependence of the groupements may be due to their interacting after having emerged asynchronously, rather than being a logical reason for expecting synchronous emergence. He believes that the connections among groupements can be derived, psychologically as well as logically, from their properties once they have been acquired. Flavell argued analogously that there is nothing in Piaget's theory that logically requires the constituent operations within each groupement to emerge in synchrony. He posutlated two explanations for Piaget's (and others') consistent finding of groupement and operational concurrence: (a) If two cognitive items emerge in an invariant order, but not too separated in time, then their true developmental relationship is not likely to be discovered empirically unless the tests used to detect them are of equivalent sensitivity with regard to their respective items; in the vast majority of cases this condition has not been met. (b) In the case of Piaget in particular, any one subject normally has participated in only one experimental task; Piaget has usually taken the average age of the subjects who gave evidence of just having acquired, say, item A, and if this average age was the same as the average age of those subjects who showed evidence of just having acquired item B, then Piaget has concluded that items A and B must emerge synchronously. Flavell correctly pointed out that drawing conclusions about within-subject phenomena from between-

subject data is not logically sound.

Empirical. Relatively few investigators have concerned themselves with Piagetian notions of time and speed. Lovell and Slater (1960) studied notions of simultaneity, the equality of synchronous durations, and temporal order as primitive components of Piaget's molar time concept. They used tasks virtually identical to those of Piaget (1971b) and obtained results not significantly different enough from Piaget's to be of any concern to the present study. Lovell, Kellett, and Moorhouse (1962) replicated much of Piaget's work on speed concepts (Piaget, 1971a), using identical or very similar tasks; their results were generally supportive of those of Piaget. Weil (1969), using Piaget's time concept tasks, found the order of difficulty of the items similar to that of Piaget's, with time, distance, and speed being confused before they become operational. Charlesworth (1962) studied the effect of rotation on a placement (linear order) task, comparing the performance of younger and older subjects. Delorme and Pinard (1970) used a modified version of a Piagetian speed concept task to study the development of notions of relative speed, which is a formal-, not a concrete-operational problem. Rothenberg (1969b) used one Piagetian speed concept task to which she added increasingly complicated variations, and found that the primary conceptual difficulty in the domain of distance is the taking into account of repeated spatial units that occur

between the starting and stopping points of a total distance travelled. In sum, none of the empirical research that has been concerned with time and speed in the Piagetian framework has focused on time and speed concepts as manifestations of groupements, and those studies concerned with operations of time and speed have been either supportive of Piaget's theories or not concerned with testing them.

A host of other studies has been concerned with other aspects of time. Ames (1946), Bradley (1947), Friedman (1944a, 1944b), Hartigan (1971), Oakden and Sturt (1922), and Schechter, Symonds, and Bernstein (1955) studied the development of children's conceptions of such things as seasons, days of the week, hours and minutes, age, the past (sense of history), and the future (making plans), while Springer (1952) studied the development of the ability to tell time and use the clock. This literature was reviewed by Jahoda (1963).

Brainerd (1972) reported the results of a series of studies of Piaget's groupement theory in the domain of simple quantification. He found that some of the groupements are acquired reliably earlier than others for all subjects; specifically, he found that all of the relational groupements are acquired earlier than any of the class groupements. Another finding was that, within each

class groupement, the inverse operation is acquired earlier than the composition operation.

The Present Study

The present study was concerned with five tasks. There was one task for the assessment of the presence or absence of each of the groupings of temporal order (groupement V), temporal interval (groupement I), spatial order (groupement V), and spatial interval (groupement I), which tested for the presence or absence of both their component operations. The fifth task was focused on the molar concepts of time and speed, testing for their presence or absence.

In Piaget's own tasks, as well as in those used in most of the empirical research cited above, the usual case has been for subjects to be questioned about, say, temporal relations which are to be derived from the observation of objects moving or water levels rising at different rates; in Piagetian tasks assessing speed operations there usually has been a similar confounding of speed with time. An effect of this kind of confounding must be to increase the probability of finding concurrence among the operations of speed and time in question, even in the absence of any true concurrence. The first four tasks of the present study were conceived so as to rigorously exclude the possibility of confounding the concepts they were to assess with any other concepts. In the case of the fifth task, it was necessary

to confound time and speed since they are defined in terms of each other at the molar level.

The following hypotheses were tested: (a) The grouping of the operations of temporal order and the grouping of the operations of temporal interval are acquired in synchrony; (b) the grouping of the operations of spatial order and the grouping of the operations of spatial interval are acquired in synchrony; (c) the temporal groupings and the spatial groupings are acquired in synchrony; (d) the composition operation and the reverse operation of each of the above groupings are acquired in synchrony; (e) the spatial operations are acquired prior to the molar concept of speed; (f) the temporal operations are acquired prior to the molar concept of time; and (g) the molar concepts of time and speed are acquired in synchrony.

METHOD

Subjects

The subjects were drawn randomly from the first-grade-, second-grade-, and third-grade-class lists of the Edmonton Public School System, and from the kindergarten-class list of a private kindergarten in a middle-class neighborhood of Edmonton.[3] The kindergarten sample was composed of 16 children (eight boys and eight girls) whose mean age was 5 years 6 months and whose age range was from 5 years 1 month to 6 years 0 months. The first-grade sample was composed of 31 children (15 boys and 16 girls) whose mean age was 6 years 7 months and whose age range was from 6 years 1 month to 7 years 3 months. The second-grade sample was composed of 31 children (15 boys and 16 girls) whose mean age was 7 years 8 months and whose age range was from 7 years 1 month to 9 years 11 months. The third-grade sample was composed of 31 children (15 boys and 16 girls) whose mean age was 8 years 9 months and whose age range was from 7 years 5 months to 9 years 11 months. A 25-year-old male served as the experimenter for all subjects.

Materials

The materials for assessing operations of temporal order

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3. It was decided to use kindergarten children for the molar time and speed task after a ceiling effect for first-grade subjects on this task was discovered.

were five petri dishes painted red, yellow, blue, black, and green; a small glass pitcher one-third full with water; a piece of white matte board 29.5 cm by 31 cm with red, yellow, and blue paper circles 10 cm in diameter glued to it such that the three circles were equidistant from each other; a similar piece of matte board with black, blue, and green circles glued to it. The materials for assessing operations of temporal interval were four soup cans with the labels removed and painted yellow, blue, black, and green; a large plastic pitcher half full with water; a piece of black cardboard 45 cm by 22 cm on which were glued two "clocks," that is, white cardboard circles without numbers, 15 cm in diameter and separated by 9 cm, each having one "hand" of black cardboard. The materials for assessing operations of spatial order were five tennis balls painted yellow, red, blue, green, and black; a wooden tunnel painted black and 35.5 cm long, 12 cm wide, and 11.5 cm in height; the same pieces of matte board with colored circles as were used for the assessment of operations of temporal order. The materials for assessing operations of spatial interval were a piece of white matte board 81 cm by 28 cm on which was glued lengthwise a "road," that is, a piece of black cardboard 71 cm by 7.5 cm, with a yellow paper circle 9.5 cm in diameter glued next to one end of the road, a blue paper circle of the same size glued next to the road 39.5 cm from the yellow circle, and a brown circle of the same size glued

next to the other end of the road, 11.5 cm from the blue circle; a similar piece of matte board with a similar road, with red, green, and black circles 9.5 cm in diameter spaced along the road as were the yellow, blue, and brown circles, respectively; a piece of white matte board 24 cm by 24 cm on which were glued replicas of the yellow, blue, and brown circles spaced equidistant from each other; a similar piece of matte board with red, green, and black circles; a metal toy car 7 cm in length. The materials for assessing the concepts of time and speed were two wooden tunnels identical to that used for the assessment of operations of spatial order; four tennis balls painted blue, red, black, and green, each skewered on the end of a 39 cm length of straight coat-hanger wire.

Procedure

All first-, second-, and third-grade subjects were tested on each of the five tasks detailed below. Kindergarten subjects were tested only on the time and speed concept task. Throughout the assessments the experimenter and the subject sat across from each other at one of two small, rectangular tables. The tasks assessing the operations of temporal order, temporal interval, and spatial order were carried out at one table, while the tasks assessing the operations of spatial interval and the concepts of time and speed were carried out at the second table. Thus, it was necessary for the experimenter and the

subject to change tables at least once in the course of the assessments, and up to four times, depending on the presentation order of the tasks. Whenever it was necessary to change tables the subject was told, "Now we have to go to the other table (again)," or, "Now we have to go back to the first table again." Only the materials for the specific task being carried out were on a table at any one time, with the exception of the two pitchers, which were judged not to be of any "play value." The remainder of the materials were kept out of the subject's view on a chair or on the floor below the tables.

Although the five assessment procedures are detailed below, it should be noted here that all of the tasks controlled for three sources of measurement error: color blindness, children's tendency to agree with what an adult says more frequently than they disagree (*cf*. Rothenberg, 1969a), and the fact that difference judgments and equivalence judgments generally are of differential difficulty (*cf*. Mehler and Bever, 1967). Color blindness was controlled by asking the subject to name the colors appropriate to each task. In the case of the temporal order, spatial order, and spatial interval tasks, the subject was asked to name the colors of the circles glued to the appropriate pieces of matte board before the task was administered. In the case of the temporal interval task, the subject was asked to name the colors of the cans as they

were placed on the table. In the case of the time and speed concept task, the subject was asked to name the colors of the tennis balls as they were placed on the table. If the task presentation order was such that the subject had previously named the colors pertinent to a given task, he was not asked again. Two subjects failed to name all colors correctly and were replaced by two other randomly selected subjects. The false-agreement effect was controlled by structuring the experimenter's questions such that it was necessary for a subject both to agree and to disagree with the experimenter's assertions to be judged as having acquired the operations in question. The differential difficulty of equivalence and difference judgments was controlled by using both questions for which the correct response was an equivalence judgment and questions for which the correct response was a difference judgment.

When the subject had entered the experimenter's room and was seated across from him at the table at which the first task was to be performed he was asked his name, which was written on the question sheet. He was then asked, "Do you like Smarties (the Canadian equivalent of M&Ms)?" After an affirmative reply, the experimenter took two Smarties from a bowl which was at all times visible to the subject, saying, "OK. Here are some Smarties. You can eat them now if you want, or you can save them for later, whatever you want to do. Now, we're going to play some games in here,

and I'll ask you some questions about them. If you watch what I do carefully and try hard to answer my questions, you can win some more Smarties. OK?" Then the first task was administered.

1. Assessment of operations of temporal order. a) The composition operation. After the color-discrimination pretest the subject was told, "We're going to play a game with some dishes, and then I'll ask you some questions. Now, watch carefully." The red petri dish was placed on the table before the subject and filled from the small pitcher while the subject was told, "I'm filling the red dish with water." The blue dish was then placed on top of the red dish and filled while the subject was told, "I filled the red dish before I filled the blue dish," with a heavy emphasis on the word "before." After a pause of several seconds to ensure that the subject got a good look at the dishes, the red dish was quickly removed from beneath the blue dish and placed out of the subject's view while the yellow dish was placed on top of the blue dish. While the yellow dish was being filled the subject was told, "Now, I filled the blue dish before I filled the yellow dish." After another pause the two dishes were removed from the table and the matte board with the red, blue, and yellow circles was immediately placed on the table in front of the subject, who was then told, "Here are pictures of the dishes." The following questions were then asked: 1) "Did I

fill the red dish *before* the yellow dish?" 2) "Did I fill the yellow dish and the red dish at the *same time*?" 3) "Was the red dish full *before* the yellow one?" 4) "Were the red and yellow dishes filled at the *same time*?" 5) "Did I put water in the red dish *before* the yellow one?" 6) "Did I fill the yellow dish *before* the red dish?" For all questions, each time a dish was mentioned the circle of the appropriate color was indicated on the matte board to the subject. Then, after the matte board had been removed, regardless of the subject's performance, he was told, "Good. Here are (or, you win) some more Smarties." He was given two Smarties. "Now we're going to do it again with some different dishes, and I'll ask some more questions." *b*) *The reciprocal operation*. The procedure was repeated with black, blue, and green dishes in that order, and the subject was told, "Now here are some more questions." 1) "Did I fill the green dish *after* the black dish?" 2) "Did I fill the black dish and the green dish at the *same time*?" 3) "Was the green dish filled *after* the black one?" 4) "Were the green and black dishes filled at the *same time*?")5 "Did I put water in the green dish *after* the black one?" 6) "Did I fill the black dish *after* the green one?" The subject was again rewarded.

2. *Assessment of operations of temporal interval*. *a*) *The composition operation*. The subject was told, "Now we're going to play a game with some cans, and when we're finished

I'll ask you some questions." The blue and yellow cans were placed on the table in a line parallel to its long axis, and the piece of cardboard with the clocks was placed between the subject and the cans such that each can was directly above a clock. The hands of the clocks were pointed to the 12:00 position relative to the subject. After the color-discrimination pretest the subject was told, "These clocks will measure the filling time for the cans." The blue can was filled completely from the large plastic pitcher, after which the subject was told, "I filled the blue can and it took this much filling time." The hand of the clock corresponding to the blue can was moved to the 30-second position. Then the yellow can was filled about one-third full, after which the subject was told, "I filled the yellow can and it took this much filling time." The hand of the clock corresponding to the yellow can was moved to the 10-second position. After a pause the subject was told, "Now I'll ask some questions." For all questions, each time a can was mentioned it was pointed out to the subject. 1) "Did it take more time to fill both cans than to fill the blue can?" 2) "Did it take the same time to fill the blue can as to fill both cans?" 3) "Did I take more time to fill both cans than the blue can?" 4) "Did I take the same time to fill the blue can as to fill both cans?" 5) "Did I take longer to fill both cans than the blue can?" 6) "Did it take longer to fill the blue can than to fill both cans?"

The subject was then rewarded and told, "Now we're going to do it again with some different cans." The clock hands were then reset to their original positions and the water in the blue and yellow cans was poured back into the pitcher. *b*) *The inverse operation*. The procedure was repeated with green and black cans, the latter can being the one to be filled last and completely. "Now here are some more questions." 1) "If I *take away* the time I filled the black can, will there be *any filling time left*?" 2) "Is the time it took to fill the black can the *same* as the time it took to fill both cans?" 3) "If the time it took to fill the black can is *gone*, will there be *any filling time left*?" 4) "Is the time it took to fill both cans the *same* as the time it took to fill the black can?" 5) "If I *take away* the time it took to fill the black can, will there be *any filling time left*?" 6) "If I *take away* the time I filled the black can, will *all the filling time be gone*?" The subject was again rewarded.

3. *Assessment of operations of spatial order*. *a*) *The composition operation*. After the color-discrimination pretest the subject was told, "Now we're going to play a game with some balls in a tunnel and when we're finished I'll ask you some questions." The tunnel was placed on the table in front of the subject, aligned such that he could not see into either end. The yellow tennis ball was placed at the entrance to the tunnel on the subject's left, with

the blue ball placed just to the left of, and contiguous to, the yellow ball. Then the subject was told "Watch carefully. The yellow ball goes in ahead of the blue ball," as the experimenter used the blue ball to push slowly the yellow ball into the tunnel and out of the subject's view. Then the red ball was placed to the left of, and contiguous to, the blue ball, which now rested just outside the mouth of the tunnel. The subject was told, "Now watch, the blue ball goes in ahead of the red ball," as both balls were slowly pushed into the tunnel and out of the subject's view. Then the matte board with the yellow, blue, and red circles was placed on the table while the subject was told, "Here are pictures of those balls, and now I'll ask some questions." For all questions, each time a ball was mentioned the circle of the appropriate color was indicated to the subject. 1) "Is the yellow ball ahead of the red ball?" 2) "Is the red ball right up even with the yellow ball?" 3) "Did I put the yellow ball ahead of the red ball?" 4) "Did I put the yellow ball right up even with the red ball?" 5) "Was the yellow ball ahead of the red ball?" 6) "Is the red ball ahead of the yellow ball?" The subject was then rewarded, and the balls in the tunnel were removed. The subject was told, "Now we're going to do it again with some different balls and I'll ask you some more questions."

b) The reciprocal operation. The procedure was repeated with green, blue, and black balls in that order. The

questions were: 1) "Is the black ball behind the green ball?" 2) "Is the green ball right up even with the black ball?" 3) "Did I put the black ball behind the green ball?" 4) "Did I put the black ball right up even with the green ball?" 5) "Was the black ball behind the green ball?" 6) "Is the green ball behind the black ball?" The subject was again rewarded.

4. Assessment of operations of spatial interval. a) The composition operation. After the color-discrimination pretest the subject was told, "Now we're going to watch a car on a road, and I'll ask you some questions." The matte board with the road and the yellow, blue, and brown circles was placed on the table in front of the subject such that the road ran parallel to the long axis of the table and the circles were between the subject and the road, the yellow circle being on the subject's left. The car was placed on the road next to the yellow circle and pushed slowly to the other end while the subject was told, "The car starts at the yellow circle. It goes down the road and it comes to a blue circle, then it keeps on going and comes to a brown circle at the end." Then the road was quickly removed from the table and replaced by the matte board with the yellow, blue, and brown circles. The questions were: 1) "Does the car go more from the yellow circle to the brown circle than from the yellow to the blue?" 2) "Does the car go the same amount from the yellow circle to the blue circle as to the

brown?" 3) "When the car gets to the blue circle, does it have to go more to get to the brown circle?" 4) "Does the car go the same amount from the yellow circle to the brown circle as to the blue circle?" 5) "Do I have to push the car more from the yellow circle to the brown circle than from the yellow to the blue?" 6) "Does the car have to go more from the yellow circle to the blue circle than from the yellow to the brown?" For all questions, each time a circle was mentioned it was indicated to the subject. The subject was rewarded and told, "Now we'll look at another road." b) The inverse operation. The procedure was repeated with the road with the red, green, and black circles. The questions were: 1) "If I take away the road from the red circle to the green circle, will there be any road left?" 2) "Is the amount of road from the red circle to the green circle the same as from the red to the black?" 3) "If the road from the red circle to the green circle is gone, will there be any road left?" 4) "Is the amount of road from the red circle to the black circle the same as from the red to the green?" 5) "If the road is gone from the red circle to the green circle, will there be any road left?" 6) "If I take away the road from the red circle to the green circle, will all the road be gone?" The subject was again rewarded.

5. Assessment of the concepts of time and speed. a) Time. The subject was told, "Now we're going to watch some

balls have a race through some tunnels."[4] The tunnels were placed side by side in front of the subject such that he could not see into either end. The subject was then told, "First we'll watch a race between these two balls (green and black). Can you tell me what colors they are? Good. Now, they'll start at this end (points to the subject's right) but they'll finish down here (to the subject's left), so we have to watch here and see what happens." The green and black tennis balls skewered on the wires were slowly pushed, each through one tunnel, the black ball emerging noticeably before the green ball. When both balls had emerged from the tunnels, they were left placed side by side at the tunnel exits, and the subject was questioned. For all questions, each time a ball was mentioned it was pointed out to the subject. 1) "Did the green ball take longer than the black ball?" 2) "Did the black ball and the green ball take the same time?" 3) "Did the green ball take more time than the black ball?" 4) "Did the green ball and the black ball take the same time?" 5) "Was the green ball later than the black ball?" 6) "Did the black ball take longer than the green ball?" The subject was then rewarded and told, "Now we'll watch another race, this time between these two balls (red and blue). Can you tell me what colors they are? Good. Watch and see what happens." b) Speed. The procedure was

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[4] Tunnels were used to cover the paths of the balls so that the subject could not observe any overtaking.

repeated, with the red ball "winning the race." The questions were: 1) "Was the red ball _faster_ than the blue ball?" 2) "Did the blue and red balls go the _same speed_?" 3) "Did the red ball go _faster_ than the blue ball?" 4) "Did the red and blue balls go the _same speed_?" 5) "Was the red ball going _faster_ than the blue ball?" 6) "Was the blue ball _faster_ than the red ball?" The subject was rewarded.

Randomizations

In order to control for order effects such as warm-up, the presentation order of the five tasks was fully randomized across subjects. The presentation order of the two parts of each task was also fully randomized. In addition, the presentation order of the six questions within each subtask was randomized with the condition that questions for which the correct answer was "Yes" and questions for which the correct answer was "No" were alternated.

RESULTS

Analysis of Variance

All correct judgments were assigned "1s" and all incorrect judgments were assigned "0s." A 3 (Grade Level) X 2 (Groupement) X 2 (Operation) X 2 (Concept Area) mixed analysis of variance was computed. The main effects that attained significance were: (*a*) the between-subjects grade level effect (*F*=16.68, *df*=2/90, *p*<.0001), (*b*) the within-subjects effect for groupement (*F*=46.85, *df*=1/90, *p*<.0001), and (*c*) the within-subjects effect for concept area (*F*=9.64, *df*=1/90, *p*<.005) (see Table 1). Concerning *a*, orthogonal polynomials comparisons revealed that all of the improvement associated with age was accounted for by the linear trend (*F*=253.92, *df*=1/28, *p*<.0001). Concerning *b*, it can be seen from Table 2 that groupement V tasks were less difficult than groupement I tasks. Concerning *c*, it also can be seen from Table 2 that speed concept tasks were less difficult than time concept tasks. In addition, the following first-order interactions attained significance: Grade Level X Concept Area (*F*=5.88, *df*=2/90, *p*<.025), Groupement X Operation (*F*=70.61, *df*=1/90, *p*<.0001), and Groupement X Concept Area (*F*=100.99, *df*=1/90, *p*<.0001) (see Table 1). Post hoc tests (Scheffe, 1953) revealed that the first interaction was due to the fact that the third-grade subjects found the speed concept tasks less difficult than

TABLE 1

ANALYSIS OF VARIANCE

SOURCE	DF	SS	MS	F
Subjects	92	371.82		
Between Subjects	2	100.55	50.27	16.679****
Error	90	271.27	3.01	
Groupement	1	103.88	103.88	46.846****
GroupementXSubjects	2	0.56	0.28	0.126
Error	90	199.56	2.22	
Operation	1	2.60	2.60	1.960
OperationXSubjects	2	3.40	1.70	1.280
Error	90	119.50	1.33	
Concept Area	1	14.54	14.54	9.639***
Concept AreaXSubjects	2	17.72	8.86	5.875*
Error	90	135.74	1.51	
GroupementXOperation	1	120.97	120.97	70.614****
GroupementXOperationX Subjects	2	8.35	4.18	2.439
Error	90	154.18	1.71	
GroupementXConcept Area	1	120.97	120.97	100.987****
GroupementXConcept AreaXSubjects	2	0.23	0.11	0.094
Error	90	107.81	1.20	
OperationXConcept Area	1	3.36	3.36	2.750
OperationXConcept AreaXSubjects	2	1.17	0.59	0.480
Error	90	109.97	1.22	
GroupementXOperationX Concept Area	1	1.21	1.21	1.070
GroupementXOperationX Concept AreaXSubjects	2	12.52	6.26	5.534**
Error	90	101.77	1.13	
Within Subjects	651	1340.00		

TSQ/N=12536.18 N=744 SST=1711.82

*$p<.025$
**$p<.01$
***$p<.005$
****$p<.0001$

TABLE 2

MEANS FOR BETWEEN- AND WITHIN-SUBJECTS SOURCES OF VARIANCE

Source		Mean
Grade Level	1	3.63
	2	4.16
	3	4.53
Groupement	I	3.73
	V	4.48
Operation	C	4.16
	R	4.05
Concept Area	T	3.97
	S	4.25

the first-grade subjects ($p<.01$), while this was not the case for the time concept tasks (see Figure 1). The second interaction resulted from the fact that in the case of groupement V the composition operation was less difficult than the reversibility operation ($p<.01$), while in the case of groupement I the reverse was true ($p<.05$); in addition, it was found that the composition operation of groupement V was less difficult than the composition operation of groupement I ($p<.01$), while there was no significant difference between the reversibility operations of the groupements (see Figure 2). The third interaction was due to the fact that the groupement I speed concept task was less difficult than the time concept task ($p<.01$), while this was not the case for the groupement V tasks; in addition, the groupement V task in the time concept area was found to be less difficult than the groupement I task ($p<.01$), while this was not the case in the speed concept area (see Figure 3). A third-order Grade Level X Groupement X Operation X Concept Area interaction was found to be significant ($F=5.53$, $df=2/90$, $p<.01$). It seems likely that this interaction was a result of the interaction of the Grade Level X Concept Area and Groupement X Operation interactions discussed above.

Ordinal Analyses

Several within-subjects ordinal analyses were conducted to determine whether or not the parametric differences in

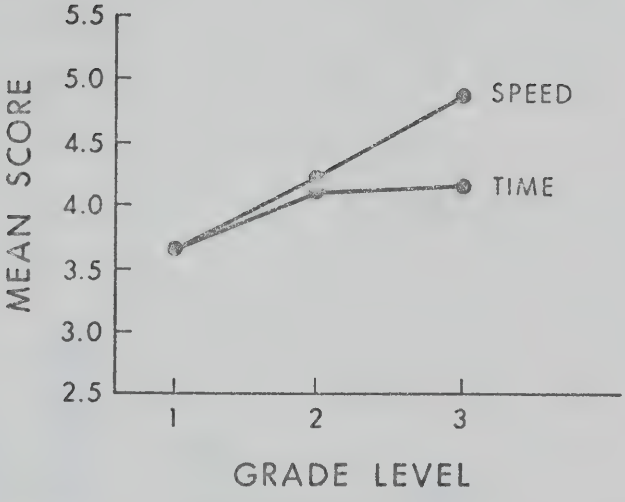

FIGURE 1. GRADE LEVEL X CONCEPT AREA INTERACTION

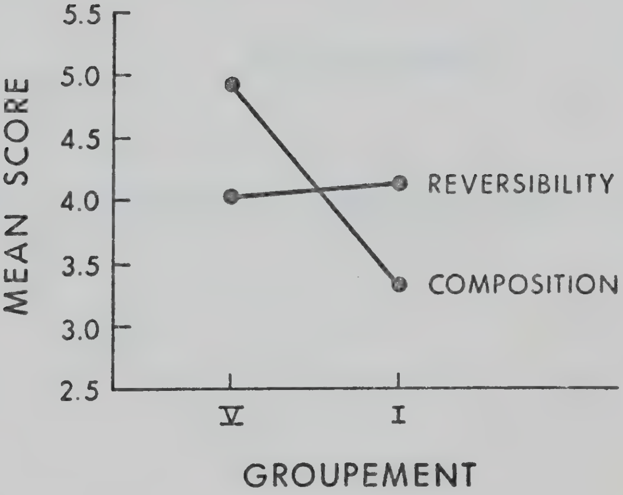

FIGURE 2. GROUPEMENT X OPERATION INTERACTION

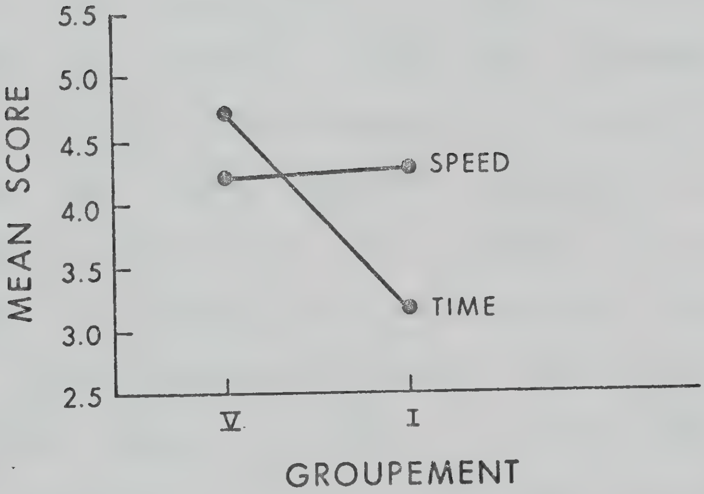

FIGURE 3. GROUPEMENT X CONCEPT AREA INTERACTION

groupement, operation, and concept area task difficulty also may be construed as invariant sequences. In addition, an ordinal analysis was computed for the molar time and speed concepts assessed by the fifth task, and the data for the kindergarten subjects were included for this analysis only.[5] Specifically, the ordinal analyses tested for invariant sequences (a) between the composition and reversibility operations for each groupement in each concept area; (b) between the two groupements in each concept area, (c) between the two concept areas; (d) between the molar concepts of time and speed assessed by the fifth task; and (e) between the time concepts assessed by the first and second tasks and the molar time concept assessed by the fifth task, and between the speed concepts assessed by the third and fourth tasks and the molar speed concept assessed by the fifth task. In order to conduct these analyses a pass-fail criterion was used to partition the subjects into those who had and those who had not acquired each of the operations or concepts in question: If the subject correctly answered five or six of the six questions for a particular operation or concept, the operation or concept was said to have been acquired; otherwise it was said not to have been acquired. If both the composition and reversibility operations of a particular groupement were acquired, the

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[5] A procedural error by the experimenter resulted in the discarding of the data of the molar time and speed concept task for one first-grade subject.

groupement was said to have been acquired; otherwise the groupement was said not to have been acquired. If both groupement I and groupement V of a particular concept area were acquired, that concept area was said to have been acquired; otherwise the concept area was said not to have been acquired. The binomial test (Siegel, 1956) was used to test the significance of the various relationships which are shown in Tables 3-6.

The probability values appearing at the bottoms of Tables 3-6 suggest that (a) in the case of groupement V the composition operation is acquired prior to the reciprocal operation, and in the case of groupement I the inverse operation is acquired prior to the composition operation, for both concept areas (see Table 3); (b) in the case of the time concept area groupement V is acquired prior to groupement I, while no conclusion can immediately be drawn concerning these groupements in the speed concept area (see Table 4); (c) the two groupements in the concept area of speed are in evidence before they are both seen in the concept area of time (see Table 5); (d) the molar concept of speed is acquired prior to the molar concept of time (also see Table 5); and (e) the molar concept of time is acquired prior to the acquisition of all of the operations of time, and the molar concept of speed is acquired prior to the acquisition of all of the operations of speed (also see Table 5). Concerning Conclusion a, for groupement V in the

TABLE 3

DEVELOPMENTAL RELATIONSHIPS BETWEEN THE COMPOSITION AND REVERSIBILITY OPERATIONS OF GROUPEMENTS I AND V IN THE CONCEPT AREAS OF TIME AND SPEED

Groupement		Groupement			
		Groupement I		Groupement V	
		Operation			
	Operation	Composition		Composition	
		Present	Absent	Present	Absent
	Time Concept Area				
	Inverse				
Groupement I	Present	1	26*		
	Absent	4*	62		
	Reciprocal				
Groupement V	Present			38	3**
	Absent			36**	16
	Speed Concept Area				
	Inverse				
Groupement I	Present	30	36**		
	Absent	4**	23		
	Reciprocal				
Groupement V	Present			29	28**
	Absent			2**	34

*$p<.001$
**$p<.0001$

TABLE 4

DEVELOPMENTAL RELATIONSHIPS BETWEEN GROUPEMENTS I AND V IN THE CONCEPT AREAS OF TIME AND SPEED

		Concept Area			
Concept Area		Time		Speed	
		Groupement			
	Groupement	Groupement I		Groupement V	
		Present	Absent	Present	Absent
	Groupement I				
Speed	Present			15	15
	Absent			14	49
	Groupement V				
Time	Present	0	37*		
	Absent	1*	55		

*$p<.00001$

TABLE 5

DEVELOPMENTAL RELATIONSHIPS BETWEEN TIME AND SPEED AND MOLAR TIME AND SPEED

Operations or Concept Area	Operation or Concept Area					
	Time Operations		Molar Time		Molar Speed	
	Present	Absent	Present	Absent	Present	Absent
Speed Operations						
Present	0	15*			15	0*
Absent	0*	78			72*	5
Molar Speed						
Present			79	21*		
Absent			1*	7		
Time Operations						
Present			0	0*		
Absent			76*	16		

*$p < .0001$

time concept area 36 subjects evidenced the acquisition of the composition operation in the absence of the reciprocal operation, while three subjects evidenced the reverse; in the speed concept area 28 subjects evidenced the acquisition of the composition operation in the absence of the reciprocal operation of this groupement, while two subjects evidenced the reverse. For groupement I in the time concept area 26 subjects evidenced the acquisition of the inverse operation in the absence of the composition operation, with four subjects evidencing the reverse; in the speed concept area 36 subjects evidenced the acquisition of the inverse operation in the absence of the composition operation, with four subjects evidencing the reverse. Concerning Conclusion _b_, in the time concept area 37 subjects evidenced the acquisition of groupement V in the absence of groupement I, with one subject evidencing the reverse; in the speed concept area 14 subjects evidenced the acquisition of groupement V in the absence of groupement I, with 15 subjects evidencing the reverse. An additional finding in this concept area was that the inverse operation of groupement I is acquired prior to the reciprocal operation of groupement V, with 42 subjects evidencing the acquisition of the former in the absence of the latter, and seven subjects evidencing the reverse (see Table 6). Seven reversals out of a population of 49, however, may be too large a proportion to warrant the conclusion that an

TABLE 6

DEVELOPMENTAL RELATIONSHIP BETWEEN THE RECIPROCAL OPERATION OF GROUPEMENT V AND THE INVERSE OPERATION OF GROUPEMENT I IN THE SPEED CONCEPT AREA

Groupement	Operation	Groupement	
		Groupement V	
		Operation	
		Reciprocal	
		Present	Absent
	Inverse		
Groupement I	Present	24	42*
	Absent	7*	20

*$p < .00001$

invariant sequence exists here. Thus, it seems reasonable to conclude that, overall, groupement V is acquired prior to groupement I. Concerning Conclusion c, 15 subjects evidenced the acquisition of both groupements in the speed concept area in the absence of their acquisition in the time concept area, with no subjects evidencing the reverse. Concerning Conclusion d, 21 subjects evidenced the acquisition of the molar speed concept in the absence of the molar time concept, with one subject evidencing the reverse. Concerning Conclusion e, 76 subjects evidenced the acquisition of the molar time concept in the absence of all of the time operations, with no subjects evidencing the reverse; 72 subjects evidenced the acquisition of the molar speed concept in the absence of all of the speed operations, with no subjects evidencing the reverse.

DISCUSSION

Groupement Theory

The principal findings with regard to groupement theory are that groupements I and V are not acquired in concurrence in the time concept area and that the composition and reversibility operations of these groupements are not acquired in concurrence in either concept area. Concerning the first finding, while it was shown that groupement V is acquired prior to groupement I in the time concept area, no asynchrony was found in the speed concept area. There are at least three possible explanations for this finding: (*a*) These groupements really are acquired in synchrony, but for some reason they were observed to emerge sequentially in the time concept area; (*b*) these groupements are acquired in a random order, that is, about half of all children approaching the concrete-operational level acquire groupement I prior to groupement V and the remainder acquire them in the reverse order, but for some reason a sequence was observed in the time concept area; (*c*) groupement V is acquired earlier than groupement I, but for some reason the sequence was obscured in the speed concept area. Concerning the first explanation, it is possible that the discrepant results were due to the temporal interval task being, in some noncognitive aspect(s) (perceptual, intuitive, abstract, information-processing, etc.), more difficult in

the context of the temporal order task than was the spatial interval task in the context of the spatial order task, in spite of the fact that the two groupements are acquired in synchrony. Concerning explanation b, it is possible that the discrepant results were due to the temporal interval task being more difficult, in some noncognitive dimension, in the context of the temporal order task than the spatial interval task was in the context of the spatial order task, in spite of the fact that the two groupements are really acquired in a random order. Given the actual data, that in the speed concept area 15 subjects evidenced the acquisition of groupement I in the absence of groupement V, while 14 subjects evidenced the reverse, and that in the time concept area the ratio was 37 to one in favor of groupement V being acquired first, it is difficult if not impossible to favor the first explanation over the second or *vice versa*. However, both of the above explanations may be faulted on two grounds. First, of the 37 subjects who evidenced the acquisition of groupement V in the absence of groupement I in the time concept area, seven were first-grade students, 14 were second-grade students, and 16 were third-grade students; the significance of this distribution is that it is extremely unlikely that such noncognitive factors as were suggested above would result in a three-year, or longer, lag between the demonstrated presence of two cognitive structures which truly emerge in synchrony or in a random

order. The second reason is that, in addition to holding across a three-year span, the sequence found in the time concept area is just too big to be dismissed by an appeal to secondary factors. Concerning explanation c, it is recalled that the primary materials used for the assessment of the operations of spatial interval consisted of "roads" with all of the colored circles about which the subjects were questioned visible simultaneously and in order. Due to the simple and perceptually undemanding layout of these materials, it is likely that concrete operations, that is, the cognitive structure which is groupement I, were not required by several of the subjects for the solution of this task; rather, preoperational intuition, the simple internalization of percepts in the form of representational images, may have been sufficient. It is accepted that the third explanation is the most reasonable and it is therefore asserted that groupement V is acquired prior to groupement I. Concerning the second finding, it was shown that in the case of groupement I the inverse operation is acquired earlier than the composition operation and in the case of groupement V the composition operation is acquired prior to the reciprocal operation, for both concept areas.

In the empirical literature only Brainerd (1972) has reported research focused directly on groupement theory. The finding that groupement V is acquired earlier than groupement I is consistent with his finding that all of the

relational groupements are acquired before any of the class groupements. The finding that for groupement I the inverse operation is acquired prior to the composition operation is consistent with Brainerd's finding that this sequence obtains for all of the class groupements. Brainerd did not report finding any sequential acquisition of the operations of the relational groupements; because his paper is a summary report the data were not presented in sufficient detail to warrant speculation as to why his findings and the present findings differ on this point.

The above findings are inconsistent with groupement theory in which it is stipulated that all of the groupements emerge synchronously in a given concept area and that all of the operations of the groupements also emerge synchronously. Brainerd (1972) has stated that both of these stipulations are such crucial empirical predictions of the theory that any contrary finding must be detrimental to its status as a model for middle-childhood thought. Pinard and Laurendeau (1969) have argued that such findings would not only have the effect of vitiating groupement theory, but also of seriously jeopardizing Piaget's entire stage concept by undermining the structuring criterion. Flavell (1970, 1971) and Flavell and Wohlwill (1969), on the other hand, have argued that there is no logical reason for the theory to require the concurrent emergence of groupements; rather, according to these authors, whatever logical and

psychological connections Piaget postulates to exist among the groupements may be formed after they all have asynchronously emerged, and the stipulation that all groupements emerge together may be a restriction without which the theory fares better. Flavell (1970, 1971) has put forth a similar argument with respect to the constituent operations of a given groupement. Brainerd (1973), however, has shown on logical grounds that if an organism possesses one of the defining traits of a given stage, it must also tend to possess the remaining traits of the stage, if that stage is to be taken to correspond to something in the real world. In the case of Piaget's theory the stage is that of concrete operations and the defining traits are the eight groupements and their 16 constituent operations. It has been shown in the present study that the presence of one defining trait (for example, groupement V or the inverse operation of groupement I or the composition operation of groupement V) is not necessarily accompanied by the tendency to possess another of the traits (groupement I or the composition operation of groupement I or the reciprocal operation of groupement V, respectively). This is the sort of evidence that is generally believed challenges groupement theory as a model for middle-childhood thought. Whether or not such evidence challenges Piaget's entire stage theory, as Pinard and Laurendeau (1969) have suggested, can only be decided by future research.

Time and Speed

Before the findings concerning Piaget's theory of the development of time and speed concepts are discussed, a brief review of the major points of the theory is in order. Piaget postulated (_a_) that for each of the groupings of temporal order, temporal interval, spatial order, and spatial interval, the composition and reverse operations are acquired in synchrony; (_b_) that the operations of temporal order, temporal interval, spatial order, and spatial interval are grouped in synchrony; (_c_) that an understanding of the molar concepts of time and speed is implied only by the presence of all of the above groupings interconnected into operations of co-displacement; (_d_) that the molar concepts of time and speed are grasped in synchrony. The findings with respect to _a_ are that in the case of the interval groupings the inverse operation is acquired prior to the composition operation and that in the case of the order groupings the composition operation is acquired earlier than the reciprocal operation. Concerning _b_, it was shown that for each concept area the order grouping is acquired earlier than the interval grouping. It was also shown that both spatial groupings are acquired before both temporal groupings, but in view of the fact that the task assessing the operations of spatial interval was probably often solved by spurious, nonoperational means, this finding is questionable. Concerning _c_, it was shown that the molar

time concept is grasped before both temporal groupings are acquired and that the molar speed concept is grasped before the spatial groupings are acquired. Concerning d, it was shown that the molar speed concept is grasped earlier than the molar time concept.

The findings to the effect that the constituent operations of the various groupings are not acquired in synchrony and that the groupings themselves are not acquired in synchrony were discussed in the previous section. The conclusion reached, that the status of groupement theory as a model for middle-childhood thought is challenged, certainly applies to the specific concept areas of time and speed. And even if the notion of grouping is discarded and just the individual operations considered, it is clear that the theory of the development of time and speed is not viable; for the molar concepts of time and speed, in addition to not having been grasped in synchrony, were grasped earlier than any of their proposed operational precursors by some subjects and earlier than at least one of their proposed precursors by most subjects.

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APPENDIX A

CELL MEANS FOR ANALYSIS OF VARIANCE

		Groupement			
		I		V	
		Operation			
Concept Area	Grade Level	Composition	Inverse	Composition	Reciprocal
	1	2.61	3.10	5.13	3.68
Time	2	2.81	3.87	5.36	4.36
	3	3.19	3.55	5.45	4.49
	1	3.68	3.74	3.61	3.49
Speed	2	3.68	4.84	4.68	3.71
	3	4.36	5.36	5.42	4.39

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